**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans:- **(B) 0.2676**

Explanation :-

The work begin after 10 min, so the average time increase from 45min to 55min.

for normal distribution :-

z = (X-μ)/б

Z = (60-55)/8

Z = 0.625

P(E) = 73.4%

Probability that the service manager will not meet his demand will be = 100-73.4 = 26.6% or 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.
3. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Ans:- **(A) False , (B) True**

Ans - We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

**A)** Probability of employees greater than age of 44= Pr(X>44)

Pr(X > 44) = 1 - Pr(X d" 44).

Z = (X -µ )/ = (X - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X d" 44) = Pr(Z d" (44 - 38)/6) = Pr(Z d" 1)=84.1345%

Probability that the employee will be greater than age of 44 = 100-84.1345=15.86%

So, the probability of number of employees between 38-44 years of age = Pr(X<44)-0.5=84.1345-0.5= 34.1345%

Therefore, the statement that More employees at the processing center are older than 44 than between 38 and 44 is **False**.

**B)** Probability of employees less than age of 30 = Pr(X<30).

Z = (X -)/ = (30 - 38)/6

Thus, the question can be answered by using the normal table to find

Pr(X d" 30) = Pr(Z d" (30 - 38)/6) = Pr(Z d" -1.333)=9.12%

So, the number of employees with probability 0.912 of them being under age 30 = 0.0912\*400=36.48(or 36 employees).

Therefore, the statement B of the question is also **TRUE**.

Ans - We have a normal distribution with = 38 and = 6. Let X be the number of employees. So according to question

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans:-

Ans= By using the properties of mean and variances.

* The mean of a sum of random variables is equal to the sum of their means.
* The variance of a sum of independent random variables is equal to the sum of their variances.

we can find the mean and variance of 2X1 and X1 + X2:

Therefore, we can see that 2X1 and X1 + X2 have the same mean of 2μ, but different variances of 4σ^2 and 2σ^2.

To discuss their distributions, we can say that both 2X1 and X1 + X2 follow normal distributions since they are linear combinations of independent normal random variables. Specifically, 2X1 follows a normal distribution with mean 2μ and standard deviation 2σ, while X1 + X2 follows a normal distribution with mean 2μ and standard deviation sqrt(2σ^2) = sqrt(2)\*σ.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans:- **(D) 48.5, 151.5**

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
3. Specify the 5th percentile of profit (in Rupees) for the company
4. Which of the two divisions has a larger probability of making a loss in a given year?

Ans:-

**(A) $2.2 million to $21.79 million, that is 99million rupees to 980.55million rupees.**

**(B) $3.775732 million, that is 169.9079 million rupees**

**(C) Second Division, that is ~N(7,4^2) has higher probability of making loss, due to higher variability.**